3 Chances Are...Understanding Probability and Risk

Overview

The concept of probability, or chance, plays an important role in risk assessment. In this activity, students will conduct a series of experiments, such as tossing coins, to develop an understanding of probability. They will then apply their knowledge of probability to a scenario about the potential risk of using cellular phones.

Background

Almost nothing in life is 100 percent certain because most events can be influenced by ***chance***. Still, we often try to predict the occurrence of events, such as the weather or the outcome of an election or sporting event, on the basis of observations and statistical calculations. A calculation of the possibility of a risk occurring is based on ***probability***. In other words, it is an estimate of what will *most likely* happen as a result of ***exposure*** to a ***hazard***. Probability theory provides a framework for evaluating the reliability of the conclusions reached and the inferences made when applying statistical techniques to the collection, analysis, and interpretation of quantitative data (Jardine and Hrudey 1997, B).

Probability can be characterized in three general ways (Jardine and Hrudey 1997, B):

- Probability can be generated from *actual* (or observed) data. For example, if we roll a pair of dice 1,000 times and record our results, we can generate a probability estimate of getting a combined value of 2, 3, 4, ..., or 12.
- Probability can be generated from experience and characterized as *frequencies*. For example, an insurance company might collect data on the frequency of occurrence of damages from a flood and might generate a probability of expected losses from this event.
- Probability can be generated from *expert judgment*. For example, a team of geologists who have been studying the accumulation of strain along a particular fault line can use that data to give a probability estimate (the likelihood) that there will be an earthquake in that area.

Ideally, risk assessments would be based on discrete events, such as the toss of a pair of dice, for which we could determine all possible outcomes and the chances of each outcome occurring. The reality of most risk assessments, however, is that they generate estimates based on either frequencies or expert judgments. These types of estimates involve a certain degree of inference because there are limits to the amount of data available. Consequently,

Subjects
Environmental Science, Health, Math (Statistics and Graphing), Physics

Concepts
- Ecosystems possess measurable indicators of environmental health. (7.4)
- Populations of organisms exhibit variations in size and structure as a result of their adaptation to their habitats. (10.1)
- Technologies vary in size, structure, and complexity and in their positive and negative effects on the environment. (11.1)
- Demographics influence environmental quality, government policy, and resource use. (12.3)

Skills
Calculating, Graphing, Organizing Information, Predicting

Objectives
Students will (1) learn how to calculate simple probabilities, (2) develop definitions for discrete random variable, continuous random variable, binomial distribution, and normal distribution, (3) learn to use graphs to visually represent binomial and normal distributions, and (4) develop an understanding of the relationship among probability calculations, uncertainty, and estimation of risk.

Materials
One coin (pennies) for each member of the class and copies of the Student Pages "Data Sheet for Coin Toss Predictions" and "Data Sheet for Actual Coin Tosses" on pages 44–45. If you decide to do the optional activity in Part A, Section 2, Step 6, you will also need one overhead transparency or copies of the Student Page "Tree Diagrams" on page 43.

Time Considerations
Preparation: 30 minutes
Activity:
Part A—one 50-minute period
Part B—30 minutes

there is always the possibility that the actual results will differ from the results predicted in a risk estimate. This possibility of error, often referred to as ***uncertainty***, is always present in risk assessment.

There are many sources of uncertainty in risk assessment, from the study design, to the collection of data, to the interpretation of the results. For example, all experimental studies are based on samples from larger populations, commonly referred to as ***parent populations***. The use of ***samples*** is one way of introducing uncertainty into a study's results; any sample will, by chance, differ at least a little from its parent population. As the sample size increases, the sample is less likely to differ from its parent population. Therefore, the probability that chance played a role in the study results is smaller.

For information and a related activity on characterizing uncertainty using ***p-values*** and ***confidence intervals***, please refer to the Extension at the end of Part B of this activity. In this extension activity, students will begin to understand the role that probability and uncertainty play in estimating environmental risk and in understanding risk studies.

Part A [1]

PROBABILITY AND TOSSING A SINGLE COIN

GETTING READY

Have at least one coin available for each member of the class.

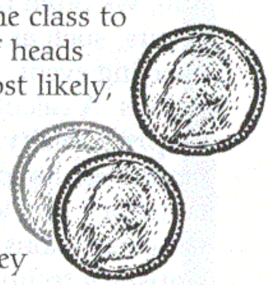
DOING THE ACTIVITY

1. Ask students if they have ever taken a chance. What kind of chances did they take? How did they determine whether or not the chance was worth taking?

2. On the chalkboard write a list of the topics that will be discussed during the activity. The list should include *probability, chance, uncertainty, sample size, predicted results*, and *actual results*. Do not, however, discuss those words at this point. As you go through the activity and the students first experience what the words mean, you can more formally guide the students to define the terms.

3. Ask the class the following question: "If you tossed a coin 10 times, how many heads and how many tails would you expect to get?" Ask several students to make a prediction. (Most high school students will probably suggest five heads and five tails.)

4. Ask for a volunteer to toss a coin 10 times. While one student flips the coin, ask the other members of the class to keep track of the number of heads and the number of tails. Most likely, this demonstration will not result in five heads and five tails. That being the case, ask the students to suggest why they got the results they achieved.



5. Discuss with the class the difference between "predicted" results and "actual" results in an experiment. Emphasize that the predicted probability of getting 50 percent heads and 50 percent tails is based on very large numbers of coin flips; the more times we would flip the coin, the closer we would come to getting the predicted results. Then tell the class that they will test the hypothesis that "the more times a coin is flipped, the closer the result will come to the predicted result of 50 percent heads and 50 percent tails."

NOTE:

You may want to point out that the number of coin flips represents the sample size in your experiment and that each coin flip is an observation.

6. Ask each student in the class to flip a coin 10 times and to keep track of his or her own results. After the students complete this task, tabulate the entire class's results. Then discuss the following questions with your students:

? How do these combined results compare with the previous results of just 10 flips?

? Was the combined result 50 percent heads and 50 percent tails? If not, ask the class members what they could do to further test the hypothesis? (Someone will most likely suggest flipping the coins more times.) You could continue this activity by having the members of the class each toss a coin another 10 times and combine the results again. At this point, you will most likely end up with a result approaching 50 percent heads and 50 percent tails, depending on the size of your class.

7. Explain to the class that it is often difficult to perform experiments with large sample sizes because of economic, time, or resource constraints. Scientists may use computer-generated observations, when applicable, to create large sample sizes. Such sampling is often accomplished using the ***Monte Carlo method***. (For more information on this method of generating observations and for an activity to explain it, refer to Appendix 8.)

Part A (2)

PROBABILITY AND TOSSING TWO COINS

GETTING READY

Make copies of the Student Page "Data Sheet for Coin Toss Predictions" for each member of the class. Optional: Make an overhead of the Student Page "Tree Diagrams."

DOING THE ACTIVITY

1. Ask the class to predict what would happen if *two* students each flipped a coin—both at the same time. How many heads or tails do they predict would result? What is the possibility of a mixed result (one head and one tail)? Ask students how they arrived at their predictions. Explore this thought process with several students before going on to the next step.

2. Ask the class members to *predict* how many heads, tails, and mixed results they would get if two students simultaneously flipped the coins 20 times. Record their predictions on the chalkboard.

3. Organize the class into pairs, and have each pair flip two coins simultaneously, recording the results for two heads, two tails, and mixed (one head and one tail) for 20 trials. Have each pair share the tally, and ask if anyone was surprised by the results. Encourage the students to explain the difference between what was expected (or predicted) and what actually occurred.

4. Explore the predictions and the results with the class. More than likely, most members of the class will have expected an approximately even distribution among the three categories and will probably be surprised when they find that the mixed result will actually occur far more often than heads or tails. It might not be so obvious to the class that the mixed category actually contains two separate outcomes: head-tail and tail-head. The following diagram technique—known as a ***tree diagram***—will help the students visualize what theoretically happens. Introducing the tree diagram here will also be useful when you do Activity 4, "Risk Assessment: Tools of the Trade," which discusses how ***event trees*** and ***fault trees*** are used as risk assessment tools.

Use Figure 3 to explain that there were actually twice as many chances for a mixed result as there were for either a two-heads or a two-tails result.

Figure 3. Tree Diagram